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DETERMINATION OF ENERGY ABSORPTION LEVEL IN THE PROCESS OF CONTACT INTERACTION OF GRINDING BODIES

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Abstract. This paper addresses the problem of quantitatively describing energy absorption during the contact interaction of grinding bodies in mills of various types. The aim of the study is to develop an analytical framework for describing the motion of grinding bodies, taking into account their dissipative properties, which are defined through dimensionless parameters, particularly the coefficient of restitution. A new approach is proposed that does not rely on the traditional Rayleigh energy dissipation formula or the classical equation of damped harmonic oscillations, but instead employs nonlinear differential equations of motion with dissipative forces proportional to the gradient of the potential energy of elastic deformation. Numerical methods, specifically the fourth-order Runge–Kutta method, are used to solve these equations. The study investigates interaction models for the most typical scenarios: collisions between two balls, a ball and a massive plate, a rod and a plate, etc. Equations of motion are derived for each of these scenarios, accounting for body geometry, type of contact, and loss coefficient. For ball collisions, the elastic contact stiffness considers the nonlinear deformation behavior described by Hertzian theory. For rod impacts, a simplified one-sided elastic connection model is applied. It is shown that as the energy loss coefficient increases, the restitution coefficient decreases, which corresponds to the physical nature of the process. The results of numerical modeling are illustrated by displacement and velocity graphs, from which the relationships between the restitution coefficient and the energy loss coefficient are also derived. It is demonstrated that in many cases this relationship approaches linearity for small loss coefficients but becomes significantly nonlinear at higher values. The obtained results have important practical significance, as they enable more accurate estimation of energy losses in mills, improve dynamic modeling of mill structures, and justify parameters for effective vibro-impact operating modes. It is noted that the majority of energy is absorbed during the contact interactions between grinding bodies themselves, even in the absence of processed material in the contact zone, which partly explains the low efficiency of milling equipment. Therefore, the findings of this study can be effectively used both in designing energy-efficient grinding equipment and in optimizing technological processes within "smart manufacturing" systems, where precise control of energy consumption and particle dynamics is essential.

Keywords: mill, grinding body, energy absorption, restitution coefficient, nonlinear differential equation, numerical methods.

1. Introduction

The processing of mining materials, including the extraction of valuable components, remains a pressing issue today [1, 2]. One of the most critical stages in processing is grinding, which typically occurs through numerous contact interactions between grinding bodies [3]. It is primarily between these grinding bodies that the fragmentation of coarse particles takes place, resulting in fine particles that, after several or dozens of cycles, form the basis of the grinding products [4]. Therefore, studying the elementary contact interactions during grinding is of significant scientific and practical interest.

Despite the fact that most impacts in mills do not lead to particle fragmentation due to the absence of particles in the contact zones of the balls, they strongly influence parameters such as energy absorption levels and the operational regimes of

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vibration mills, particularly in systems like the “grinding chamber – technological load” [5].

Numerous studies on grinding examine the collisions of grinding bodies, such as balls, in elastic formulations. For example, [6] applies the classical Hertz theory of elastic ball collisions, which proposes a nonlinear relationship between contact force and deformation [7, 8]. However, this theory does not account for the dissipative properties of materials.

To address this issue, the traditional approach involves using a differential equation of mutual displacement (approximation of centers of mass) for contacting bodies, based on the classical second-order linear differential equation of dynamics [9].

Today, for a more realistic description of oscillations in complex systems, especially those involving nonlinearities or impact interactions, nonlinear models are employed, such as the Duffing-type equation [10]:

$$M\ddot{y} + \mu\dot{y} + Cy + \alpha y^3 = 0, \quad (1)$$

where y is the displacement, m; M is the reduced mass of contacting bodies, kg; μ is the dissipation coefficient, (N·s/m); C is the contact stiffness, N/m; α is the proportionality coefficient of the nonlinear term, N/m^3.

Without the last term, the equation reduces to a linear second-order differential equation for the classical Rayleigh dissipation model [9].

However, such models have a significant drawback: they cannot explain the constant value of the coefficient of restitution (*COR*) [11], e.g., for a ball colliding with a massive plate. The *COR* is defined as the negative ratio of post-impact velocity υ_1 to initial velocity υ_0:

$$k = COR = -\frac{\upsilon_1}{\upsilon_0}. \quad (2)$$

For steel balls and plates, our experimental results show that *COR* is approximately 0.8–0.85. Also, it remains independent of the size, material, or initial impact velocity, as long as contact deformations stay within the elastic range (no plastic deformation). This corresponds to the range of contact velocities typical for the vast majority of grinding equipment types.

This is also confirmed by experimental studies [12] and [13], which determined that for steel balls the *COR* remains constant at 0.8–0.9 under relatively low impact velocities in the elastic zone, and is practically independent of the mass and radius of the balls.

In study [14], using different types of particle packing in granular materials, it was experimentally demonstrated that the *COR* increases with denser packing and lower impact velocity.

However, these facts cannot be confirmed using an equation of the type (1), since at the onset of deformation the dissipative component of the reaction force $(\mu\dot{y})$ becomes sharply and unrealistically large even for infinitesimally small deformations, while the elastic component (Cy) is still infinitesimally small, contradicting the physical meaning of the process.

Therefore, even if one manages to compute the *COR* for a single half-cycle of oscillation during contact, based on reduced mass, dissipation coefficient, contact stiffness, and initial velocity, it must be recalculated anew for each different case.

Moreover, Hertzian theory provides a clear conclusion regarding the nonlinear dependence of the contact force on deformation for spherical bodies, something that cannot be captured by the classical equation.

The second drawback can seemingly be addressed by transitioning to equation (1), but this still does not resolve the primary issue concerning the constancy of the *COR*.

In our view, the only justification for the widespread use of the classical Rayleigh dissipation model is the possibility of obtaining an analytical solution to the equation of motion [15].

However, today the rapid development of computational technology and numerical methods for solving differential equations makes the continued application of classical approaches impractical, favoring instead modern, accurate computational methods that better reflect the physical nature of contact interactions between grinding bodies.

Among nonlinear models, study [16] is noteworthy, where an attempt was made to extend Hertzian theory by incorporating damping:

$$M\ddot{y} + \mu y^{1/2}\dot{y} + Cy^{3/2} = 0 . \qquad (3)$$

Study [17] presents the following formula for the dissipative force in the case of nonlinear damping:

$$F_{dis} = \mu y^{n}\dot{y} \text{ N}, \qquad (4)$$

where $n > 0$, typically, n values from 0.5 to 1.5 are used.

Other variations on Hertzian theory are also encountered. For example, the contact force between two particles can be calculated as follows [18]:

$$F = Cy^{3/2}\left(1 + A\dot{y}\right) \text{ N}, \qquad (5)$$

where A is the proportionality coefficient, s/m.

If adhesion is present between the particles, the contact force may be defined as [19]:

$$F = \frac{4}{3}E'R^{1/2}y^{3/2} - 3\pi\omega R + \sqrt{6\pi\omega RE'y} \text{ N}, \qquad (6)$$

where E' is the effective Young's modulus (accounting for both materials), Pa; R is the effective radius of curvature of the bodies, m; ω is the specific surface adhesion energy, J/m^2.

Study [20] generalizes various collision models and focuses on how energy loss depends on the specific form of the contact force.

Thus, works [16–20] reflect the nonlinear nature of contact forces and account for the presence of damping, but they do not provide a means to analytically justify the independence of the restitution coefficient from mass, particle radius, and impact velocity.

In view of the above, the objective of this study is to derive analytical equations of motion for grinding bodies under forceful contact with each other or with the milling chamber/drum, where the magnitude of the dissipative force depends on dimensionless energy absorption parameters that are solely linked to the coefficient of restitution and are independent of dimensional quantities.

The core idea of this study is to find a definitive relationship between the dimensionless energy loss coefficient for grinding body contacts and the dimensionless coefficient of restitution during impact.

2. Methods

The equations of motion for grinding bodies during contact interactions were derived using principles of theoretical mechanics and solid body mechanics, including Hertzian contact theory for elastic bodies. Numerical methods were applied to solve the nonlinear differential equations of motion of the grinding bodies, in particular, the fourth-order Runge–Kutta method.

3. Results and discussion

Modern mills feature a variety of designs and principles for the movement of their working elements. Among the main types of mills with grinding bodies are drum-type rotating mills, vibratory mills, and planetary mills.

In this study, we focus on the cases of interactions both between the grinding bodies themselves and between grinding bodies and the mill chamber, which is usually assumed to be a massive and flat (i.e., with a much larger radius compared to the grinding body) obstacle.

The following types of interactions are considered, taking into account the geometrical features of the grinding bodies and the trajectories of their motion:

- collision between two balls (relevant for all types of ball mills, inside the milling load);
- collision of a ball with a massive flat plate (similarly, at the interface between the load and the mill chamber);
- end-on collision of a rod with a massive flat plate (vibratory mills with vertical rods);
- radial collision between two rods (rod mills, within the milling load);
- radial collision of a rod with a massive plate (rod mills, at the interface between

the load and the drum).

It should be noted that we consider interactions only between grinding bodies and between grinding bodies and the mill chamber or drum, without taking into account the presence of crushed material in the contact zone. On the one hand, this simplifies the formulation of the problem, and on the other hand, the vast majority of grinding body contacts in a mill occur without any material particles in the contact zone — which partly explains the generally low efficiency of grinding equipment compared to, for example, crushers. Nonetheless, the motion energy of grinding bodies is absorbed by all contact impacts, regardless of whether material is present or not. Therefore, we define the dissipative properties of the mill load consisting exclusively of grinding bodies.

Determination of the absorption coefficient in the end-on collision of a rod with a massive rigid plate.

Figure 1 presents the impact scheme. A rod with height *(2H)* and diameter D (in meters) strikes a massive rigid plate with an initial velocity υ_0. At the moment of impact, the center of mass of the rod is located at a height H (in meters) above the surface of the plate, and this level is taken as the reference point for measuring the displacement y (in meters) of the rod's center of mass.

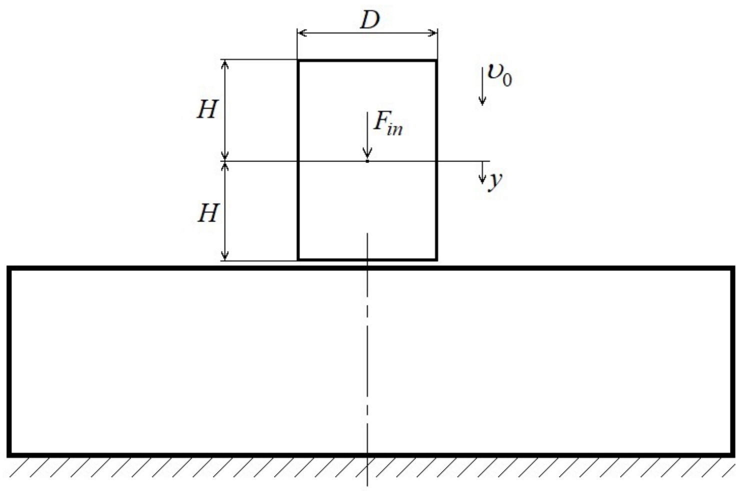


Figure 1 – Scheme of an end-on collision of a rod with a plate

Assuming simplistically that the inertial force F_{in} reduced to the center of mass compresses only the lower part of the rod, the stiffness of the rod under this deformation scheme is given by the formula:

$$C = E\frac{\pi D^2}{4H} \text{ N/m}, \tag{7}$$

where E is the Young's modulus of the rod material, Pa.

The elastic (potential) energy of deformation in the rod is given by:

$$W = \frac{Cy^2}{2} \text{ J.} \tag{8}$$

We assume that the work of the dissipative force during an elementary deformation is proportional to the change in elastic deformation energy and is always directed opposite to the motion of the system:

$$F_{dis} \cdot dy = \eta \cdot dW \cdot sign(\dot{y}), \tag{9}$$

where η is the energy loss coefficient;

$$F_{dis} = \eta \cdot \frac{dW}{dy} \cdot sign(\dot{y}); \tag{10}$$

$$F_{dis} = \eta Cy \cdot sign(\dot{y}). \tag{11}$$

Therefore, the equation of motion for the system with a rod-based contact takes the form:

$$M\ddot{y} + Cy[1 + \eta \cdot sign(\dot{y})] = 0. \tag{12}$$

This equation is derived for a one-sided elastic connection that acts only in compression, under the condition:

$$y \geq 0. \tag{13}$$

For the case of a two-sided elastic connection, to compare the trajectory shape with that of traditional damped harmonic oscillations, equation (12) takes the form:

$$M\ddot{y} + C[y + |y| \cdot \eta \cdot sign(\dot{y})] = 0. \tag{14}$$

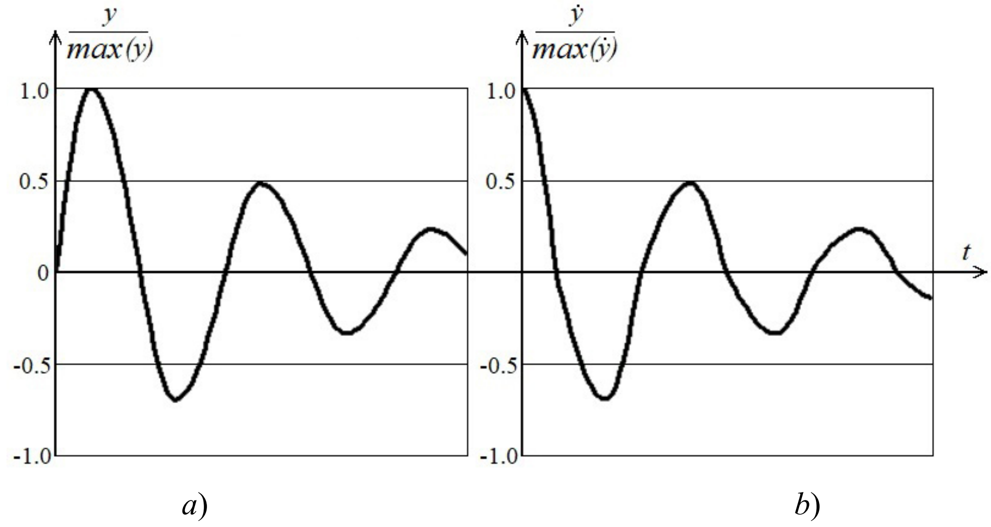
The results of the numerical solution of this equation using the fourth-order Runge–Kutta method are illustrated in Figure 2.

As can be seen, the displacement trajectory only slightly differs from that of damped harmonic oscillations, but the velocity graph shows more significant differences.

From the velocity graph, the coefficient of restitution after impact can be determined using expression (2).

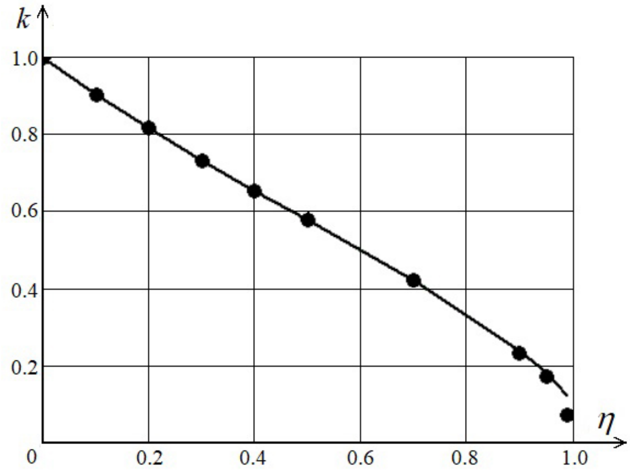
In this case, the integration step was set as $\Delta t = 0.001 \cdot T$, where T is the oscillation period, s. According to the properties of the fourth-order Runge–Kutta method, the error per integration step is of the order $O(\Delta t^5)$, and the total accumulated error at the end of the interval is $O(\Delta t^4)$, which ensures high computational accuracy [21].

It is also of interest here to examine how the restitution coefficient depends on the energy loss coefficient. The corresponding dependency is shown in Figure 3 (solid line).



a is the displacement; *b* is the velocity

Figure 2 – Calculation results for the end-on collision of a rod with a plate (with a conditional continuation of oscillations as for a two-sided elastic connection), with a loss coefficient of $\eta = 0.35$



solid line is the end-on rod collision with a plate;
dots are the ball impact with a plate (or with another ball)

Figure 3 – Dependence of the restitution coefficient on the energy loss coefficient

Determination of the absorption coefficient for radial rod collisions.

As can be seen from the graph, the greater the energy loss coefficient, the lower the restitution coefficient, which aligns with the physical meaning of the process. For small values of the loss coefficient, the dependence of the restitution coefficient tends to be nearly linear. At higher values, the relationship becomes slightly nonlinear.

A purely radial rod collision is only possible in mills that use long rods whose length is significantly greater than their diameter, specifically, in drum-type rotating rod mills. A strictly radial impact implies symmetrical compressive loading along the entire length of the rods, with their axes being perfectly parallel. In practice, such cases are nearly impossible, even in the absence of material in the drum, and therefore will not be considered further.

Determination of the absorption coefficient for ball impacts.

To derive the equation for the approach of two colliding balls, we use Hertzian contact theory [7].

In this case, the contact stiffness is nonlinear and defined by the formula:

$$C_n = a\frac{E\sqrt{D}}{\left(1-\nu^2\right)} \text{ N/m}^{1.5}, \tag{15}$$

where D is the radius of the balls, m; E is the Young's modulus of the ball material, Pa; ν is the Poisson's ratio of the ball material; a is the proportionality coefficient depending on the type of contact, namely:

- for the contact between two balls of equal radius and elastic properties:

$$a = \frac{1}{3}; \tag{16}$$

- for the contact between a ball and a massive plate:

$$a = \frac{\sqrt{2}}{3}. \tag{17}$$

The potential energy of elastic deformation for ball contacts is given by:

$$W = \frac{C_n y^{2.5}}{2.5} \text{ J}. \tag{18}$$

An analog of equation (11) (dissipative force) takes the following form:

$$F_{dis} = \eta C_n |y|^{1.5} \cdot sign(\dot{y}) \text{ N}. \tag{19}$$

Here, displacement is taken as a modulus, since only positive values can be raised to a non-integer power.

The equation of motion for spherical contacts is:

$$M\ddot{y} + C|y|^{1.5}\left[sign(y) + \eta \cdot sign(\dot{y})\right] = 0. \tag{20}$$

The expression for the elastic force with the function $sign(y)$ is:

$$F_{el} = C_n |y|^{1.5} \cdot sign(y) \text{ N}, \tag{21}$$

since the modulus of displacement is always positive, and the elastic force must be negative when the displacement is negative.

The nature of the displacement and velocity curves for ball approach is similar to that shown in Figure 2.

The dependence of the restitution coefficient for ball impacts on the energy loss coefficient is shown in Figure 3 (dots). It almost completely coincides with the corresponding dependence for end-on rod impacts (solid line).

Applications of the research results.

The results of this study can be primarily applied in the design of grinding equipment, particularly drum-type rotating and vibratory mills.

The level of energy absorption in the milling load directly influences the outcomes of dynamic calculations of mill structures. Furthermore, the ability to ensure the existence of specific vibro-impact interaction regimes—most relevant for processing hard and ultra-hard materials, as well as for producing micron-sized particles—strongly depends on the accurate determination of both the elastic and dissipative parameters of the mill's technological load.

The major part of this load typically consists not of the material being ground, but of the grinding bodies themselves, and it is precisely their dissipative properties that most significantly affect the overall energy absorption level.

Another important aspect of applying the findings is the potential to increase the energy efficiency (coefficient of performance) of the mill by accounting for and controlling the energy absorption level within the grinding load.

All of this enables the integration of grinding equipment into smart manufacturing systems and automated production lines.

4. Conclusions

1. An analytical framework has been developed for determining dimensionless parameters that account for the dissipative properties of the grinding media load in mills. Unlike traditional approaches, this method is based on the use of nonlinear differential equations and numerical techniques for their solution.

2. A new nonlinear equation of motion is proposed for grinding bodies during contact interactions, depending on the type of contact. The study covers interactions between rods and balls, both with each other and with a relatively massive mill chamber or drum.

3. A hypothesis is substantiated that the dissipative force during contact interaction between grinding bodies is proportional to the product of a dimensionless energy loss coefficient and the gradient of the elastic deformation energy at the contact.

4. In the present study, the analytical model is limited to the first half-cycle of oscillation during contact between two grinding bodies. However, it can be extended to a greater number of oscillation cycles and other types of elastic connections between bodies with constant mechanical characteristics across a wide range of

energy loss coefficients.

5. The results of this work are planned to be used in further studies of the dynamics of grinding system components, to identify operational domains of vibro-impact interaction regimes between grinding bodies, and to improve the energy efficiency of milling equipment.

Conflict of interest

Authors state no conflict of interest.

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ВИЗНАЧЕННЯ РІВНЯ ПОГЛИНАННЯ ЕНЕРГІЇ В ПРОЦЕСІ КОНТАКТНОЇ ВЗАЄМОДІЇ ПОДРІБНЮВАЛЬНИХ ТІЛ

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Анотація. У статті розглянуто проблему кількісного опису поглинання енергії при контактній взаємодії подрібнювальних тіл у млинах різного типу. Мета дослідження полягає в розробленні аналітичного апарату для опису руху помольних тіл з урахуванням дисипативних властивостей, які визначаються безрозмірними параметрами, зокрема, коефіцієнтом відновлення швидкості при ударі. Пропонується новий підхід, що базується не на традиційній формулі дисипації енергії Релея та класичному рівнянні затухаючих гармонічних коливань, а на використанні нелінійних диференціальних рівнянь руху із дисипативними силами, пропорційною градієнту потенційної енергії пружних деформацій. Для розв’язання таких рівнянь застосовано чисельні методи, зокрема, метод Рунге-Кутта четвертого порядку. У роботі досліджено моделі взаємодії для найтиповіших випадків: удару двох куль, кулі об масивну плиту, стрижня об плиту тощо. Виведено рівняння руху для кожного з цих сценаріїв з урахуванням геометрії тіл, типу контакту та коефіцієнта втрат. При цьому жорсткість пружного контакту для удару куль враховує нелінійний характер деформацій відповідно до теорії Герца. Для удару стрижня застосовано спрощену модель однобічного пружного зв’язку. Показано, що при зростанні коефіцієнта втрат коефіцієнт відновлення швидкості при ударі зменшується, що відповідає фізичному змісту процесу. Результати чисельного моделювання проілюстровано графіками переміщення та швидкості, з яких також отримано залежності між коефіцієнтом відновлення та коефіцієнтом втрат. Доведено, що у багатьох випадках ця залежність наближається до лінійної для малих значень коефіцієнта втрат, однак згодом стає суттєво нелінійною. Отримані результати мають важливе практичне значення, оскільки дозволяють точніше визначити рівень енергетичних втрат у млинах, покращити моделі динамічного розрахунку їх конструкцій та обґрунтовано встановити параметри ефективного віброударного режиму. Зазначено, що більша частина енергії поглинається саме під час контактної взаємодії помольних тіл між собою, навіть за відсутності оброблюваного матеріалу в зоні контакту, що частково пояснює низький коефіцієнт корисної дії млинів. Таким чином, результати дослідження можуть бути ефективно використані як у сфері проектування енергоефективного подрібнювального обладнання, так і для оптимізації технологічних процесів у системах "розумного виробництва", де важливим є точний контроль над енергоспоживанням та динамікою механічної взаємодії частинок.

Ключові слова: млин, подрібнювальне тіло, поглинання енергії, коефіцієнт відновлення, нелінійне диференційне рівняння, чисельні методи.